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Stability of Leadership in Bottom-up Hierarchical Organizations

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Abstract

The stability of a leadership against a growing internal opposition is studied in bottom-up hierarchical organizations. Using a very simple model with bottom-up majority rule voting, the dynamics of power distribution at the various hierarchical levels is calculated within a probabilistic framework. Given a leadership at the top, the opposition weight from the hierarchy bottom is shown to fall off quickly while climbing up the hierarchy. It reaches zero after only a few hierarchical levels. Indeed the voting process is found to obey a threshold dynamics with a deterministic top outcome. Accordingly the leadership may stay stable against very large amplitude increases in the opposition at the bottom level. An opposition can thus grow steadily from few percent up to seventy seven percent with not one a single change at the elected top level. However and in contrast, from one election to another, in the vicinity of the threshold, less than one percent additional shift at the bottom level can drive a drastic and brutal change at the top. The opposition topples the current leadership at once. In addition to analytical formulas, results from a large scale simulation are presented. The results may shed a new light on management architectures as well as on alert systems. They could also provide some different insight on last century's Eastern European communist collapse.

Keywords: local majority rules, bottom-up hierarchies, statistical physics

1. Introduction

Many human systems are organized within pyramidal hierarchies including large corporations, universities, armies, trade unions or political parties (Black, 1972). While many structural organizational frames are possible in practice, two extreme and opposite cases can be formally singled out as follows. On the one side stands the dictatorship pyramid initiated at the top of the hierarchy. Decisions go down towards the base level by level. It is totally directed and deterministic with a top-down splitting dynamics. On the other side is the perfectly democratic pyramid built up also level by level, but now from the bottom upwards to the top. The dynamics is bottom-up aggregating with the use of local

Model introduction

- Number of agents - r^N with opinions from given number of options.
- r – size of a group.
- N – number of voting steps.

Model introduction

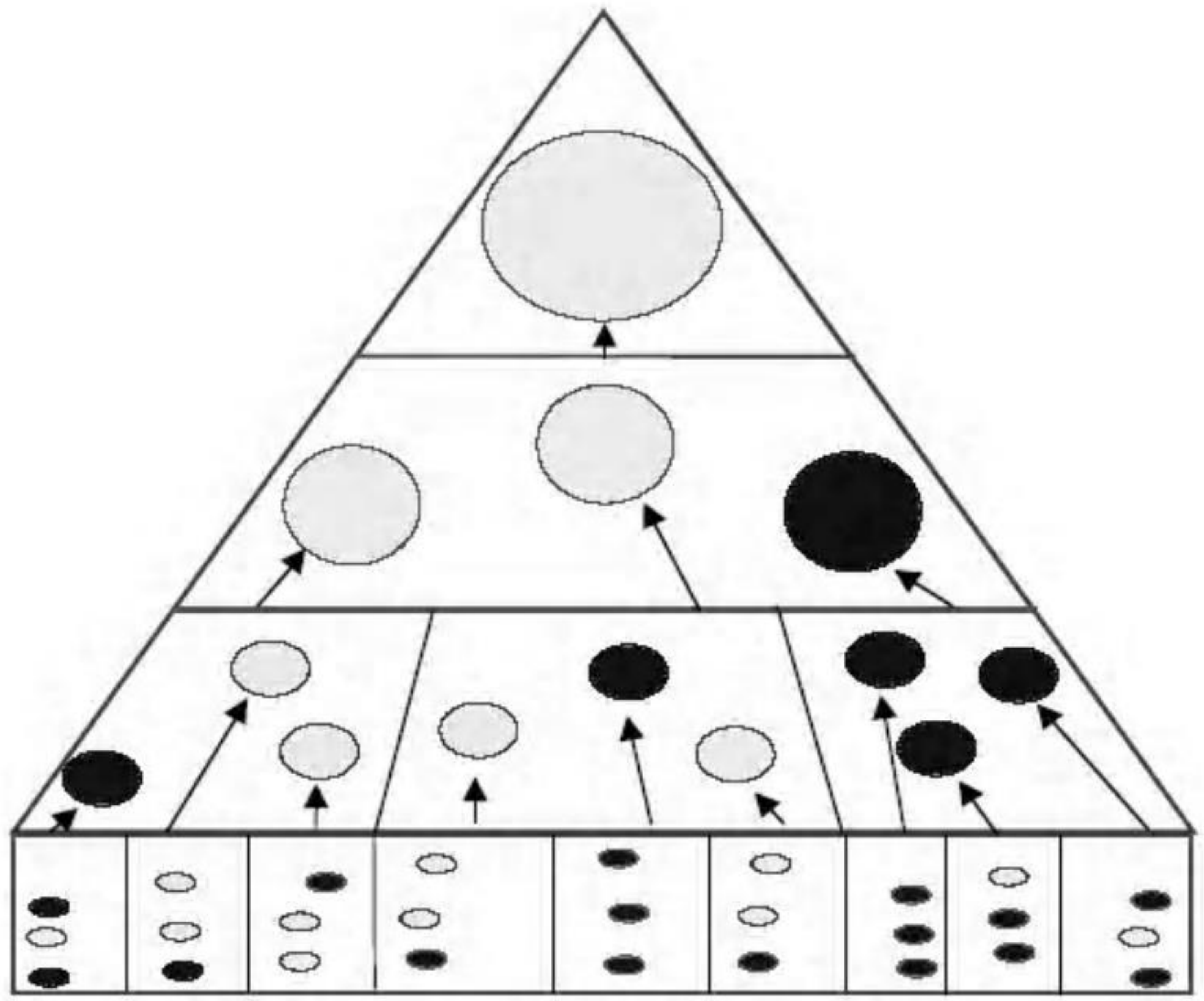

Figure 1

A two-level bottom-up hierarchy with groups of size 3

Case of odd-sized groups

- $\mathrm{r} = 3.$
- p_n - probability that an agent has opinion A at hierarchy level n.
- Probability of promoting an agent with opinion A to hierarchy level n+1.

$$p_{n+1} \equiv P_3(p_0) = p_n^3 + 3p_n^2(1 - p_n)$$

- Fixed points of this equation: $p_l = 0, p_{c,3} = \frac{1}{2}, p_L = 1.$
- p_l, p_L - stable, $p_{c,3}$ - unstable.

Case of odd-sized groups

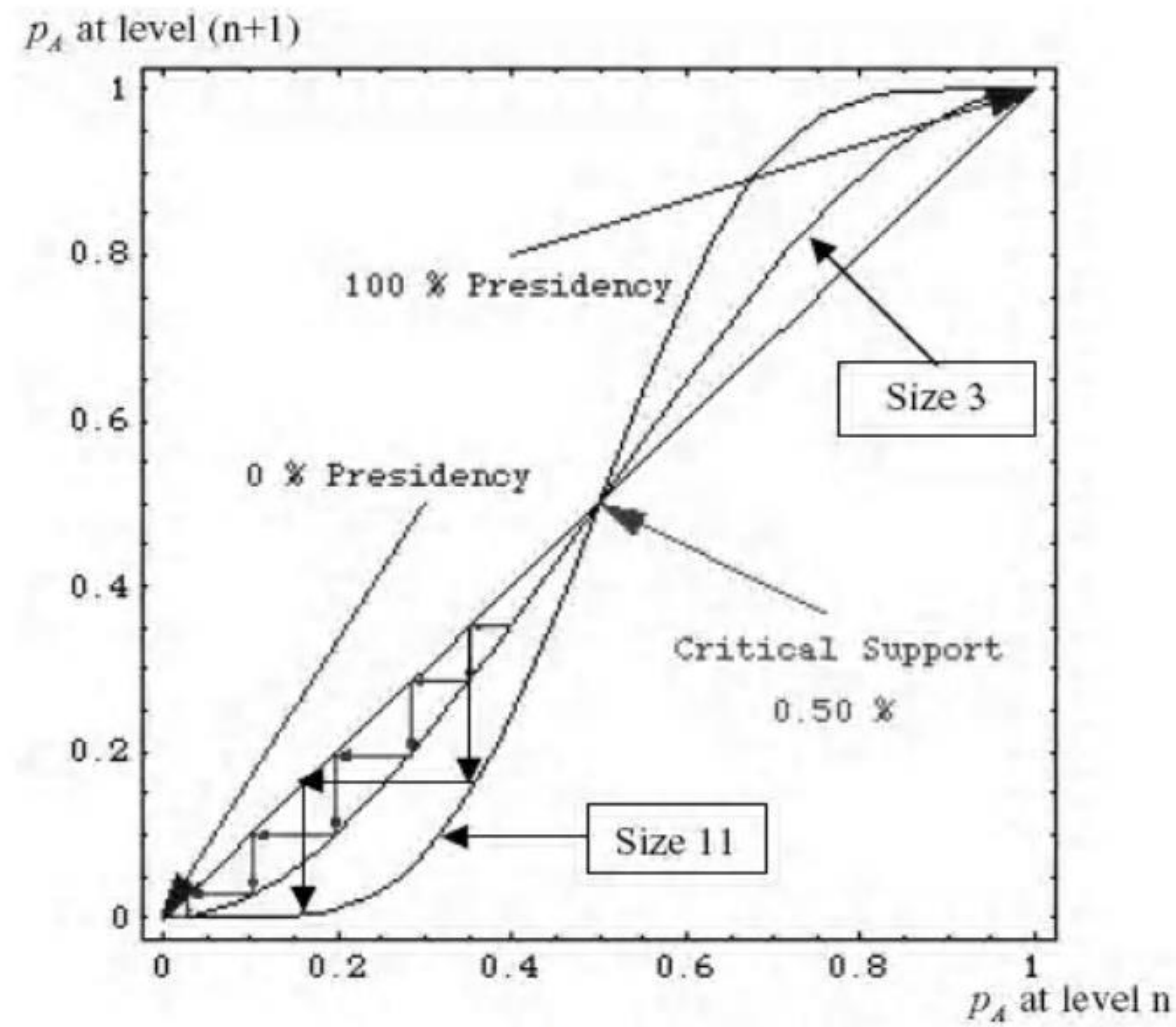

Figure 2

Variation of p_{n+1} as function of p_n for groups of respective sizes 3 and 11 with both $p_{c,3}$ = ½. Arrows show the direction of the flow.

Case of even-sized groups

- r = 4.
- Introducing bias towards opinion B.

$$p_{n+1} \equiv P_4(p_n) = p_n^4 + 4p_n^3(1 - p_n)$$

$$1 - P_4(p_n) = (1 - p_n)^4 + 4(1 - p_n)^3 p_n + 2(1 - p_n)^2 p_n^2$$

- Stable fixed points remain the same, but unstable shifts.

$$p_{c,4} = \frac{1 + \sqrt{13}}{6} \approx 0{,}77$$

Case of even-sized groups

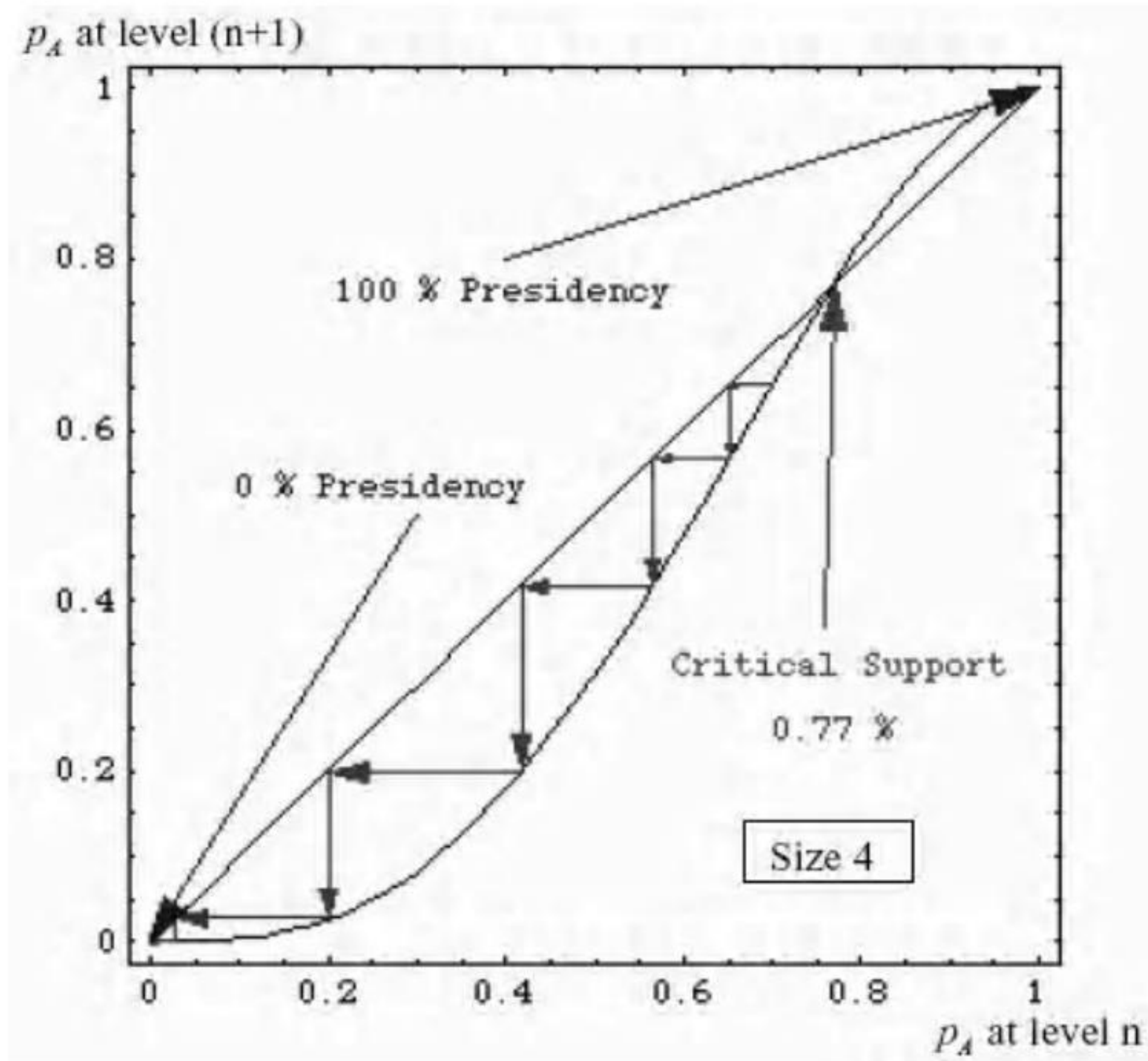

Figure 4

Variation of p_{n+1} as function of p_n for groups of respective sizes 4 with $p_{c,3} = \frac{1+\sqrt{13}}{6} \approx 0.77$. Arrows show the direction of the flow.

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Larger size groups

$$P_r(p_n) = \sum_{l=r}^{m} \binom{r}{l} p_n^l (1 - p_n)^{r-l}$$

$$m = \frac{r+1}{2}$$

For odd sizes $p_{c,r} = 0{,}5$ always. For even sizes $p_{c,r}$ decreases asymptotically towards 0,5 for $r \to \infty$.

Critical number of hierarchical levels

How many voting steps are needed to fully remove one of opinions?

$$p_n \approx p_{c,r} + \left(p_{n-1} - p_{c,r}\right) k_r$$

$$k_r \equiv \left. \frac{\partial P_r(p_n)}{\partial p_n} \right|_{p_{c,r}}$$

$$P_r\left(p_{c,r}\right) = p_{c,r}$$

Critical number of hierarchical levels

- For opinion A to win ($p_L = 1$):

$$n_c^L \approx \frac{1}{k_r} \ln \frac{p_{c,r} - 1}{p_{c,r} - p_0}$$

- For opinion B to win ($p_l = 0$):

$$n_c^l \approx \frac{1}{k_r} \ln \frac{p_{c,r}}{p_{c,r} - p_0}$$

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Critical number of hierarchical levels

Let us invert the problem – given n levels what is the minimum overall support to get full power for sure?

$$p_0 = p_{c,r} + (p_n - p_{c,r})k_r^{-n}$$

- For opinion B to win:

$$p_{l,r}^n = p_{c,r}(1 - k_r^{-n})$$

- For opinion A to win:

$$p_{L,r}^n = p_{l,r}^n + k_r^{-n}$$

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Conclusions

- Model shows how introduction of bias has much larger impact in smaller groups.
- For larger numer of starting agents there is no way for minority to win.
- Model can be considered for explaining collapse of Eastern European communist parties.

Thank you for listening

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